First Semester Examination
2017/2018 Academic Session

January 2018

EAS663 – Dynamics and Stability of Structures

Duration : 2 hours

Please check that this examination paper consists of EIGHT (8) pages of printed material including appendix before you begin the examination.

Instructions: This paper contains **SIX (6)** questions. Answer **FOUR (4)** questions.

All question must be answer in English

Each question **MUST BE** answered on a new page.

1. (a) Plot deformation response curve against frequency ratio (frequency-response curve) for a damped system with damping ratio of 0.1, 0.2 and 1.0 under harmonic excitation. Explain the characteristics of the plot at different frequency ratios ($\omega/\omega_n << 1$, $\omega/\omega_n >> 1$ and $\omega/\omega_n = 1$) and determine the parameter (mass, damping or stiffness) that controls the dynamic deformation for each case.

[10 marks]

(b) A single degree of freedom system is excited by a simply harmonic force as shown in **Figure 1**. Assume the girder is rigid whereas the columns are flexible to the lateral deformation but rigid in vertical direction. Using E = 200 GPa and I = 100(10^6) mm^4, 5 % damping and neglecting the mass of columns, determine

(i) the natural period of building,
(ii) the steady state amplitude of vibration, and
(iii) the maximum shear force and bending moment in the column.

[15 marks]

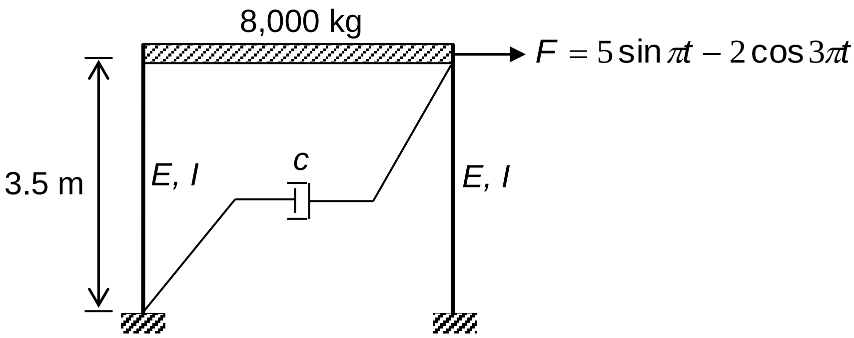


Figure 1

2. Formulate the equations of motion for a N-storey building excited by ground motion $\ddot{u}_g(t)$ as shown in **Figure 2**. State all assumptions made. Explain the way to determine the natural frequencies and vibration mode shapes. Solve the equations of motion to get the response of the system. Show in your answer how to uncouple the equation of motion.

[25 marks]

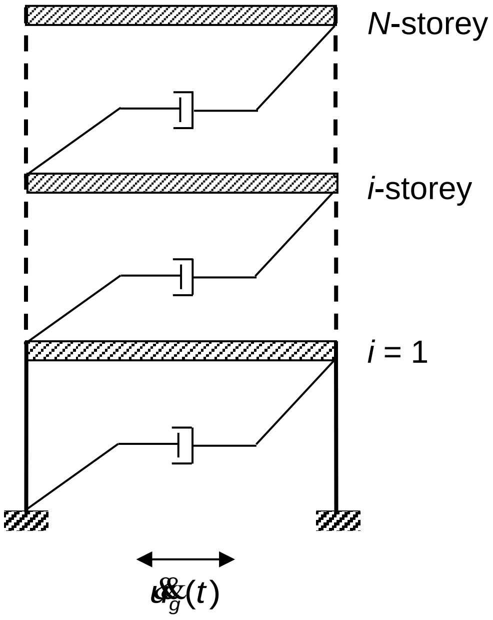


Figure 2

3. (a) Determine the critical load and effective length for the column shown in **Figure 3** using method of neutral equilibrium. The lower end of the column is pinned and the upper end is prevented from rotating but free to translate laterally. Given flexural rigidity of the column is *EI*.

[10 marks]

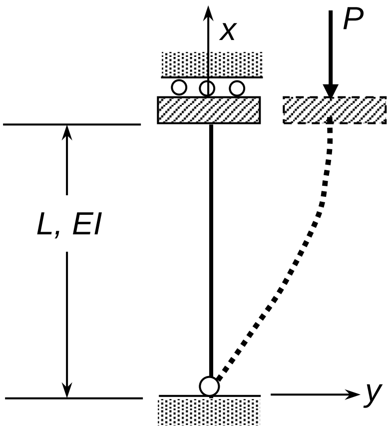


Figure 3

(b) The second order differential equation for the imperfect column shown in **Figure 4** is given as follows:

$$EIy'' + Py = -Pe$$

where *EI* : flexural rigidity of the column and *e* : eccentricity of the load *P*.

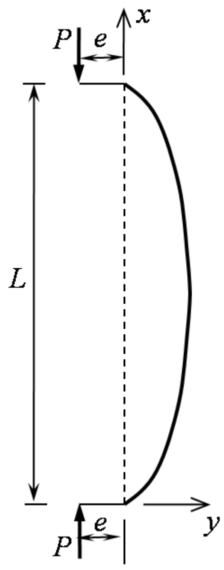


Figure 4

Using the above equation, derive the following relation between mid-height deflection δ and P/P_E:

$$\delta = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_E}}\right) - 1\right]$$

where P_E : Euler buckling load ($=\pi^2 EI/L^2$) of the corresponding perfect column.

Using appropriate sketch based on the above derived equation, explain the important differences in behavior between imperfect column and perfect column.

[15 marks]

4. (a) Obtain the critical load of the cantilever stepped column as shown in **Figure 5** using Rayleigh-Ritz method. Assume the following deflected shape:

$$y = ax^2 + bx^3$$

where *a* and *b* : unknown coefficients.The member is made of material with elastic modulus *E*. Suggest one way to improve the accuracy of the critical load estimated using Rayleigh-Ritz mehod.

[17 marks]

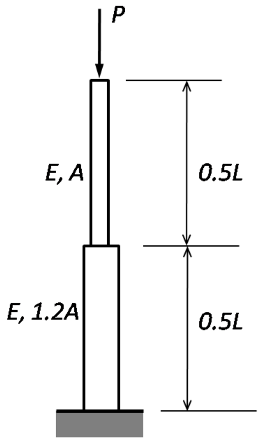


Figure 5

(b) Using the example of simply supported beam-column member as shown in **Figure 6** subjected simultaneously to a point load *W* at mid-span and an axial load *P*, explain the amplification effect on the mid-span deflection due to the presence of *P*. Use suitable equation in your explanation. Derivation of the equation is not needed.

[8 marks]

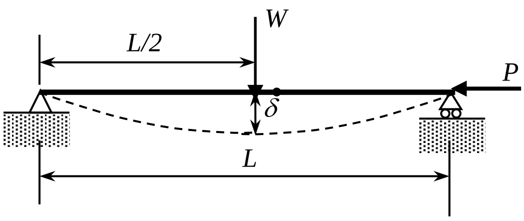


Figure 6

5. (a) Using slope-deflection equation, determine the critical load and the corresponding effective length for column AB as shown in **Figure 7.** Refer to Appendix A for the corresponding slope-deflection equations.

[17 marks]

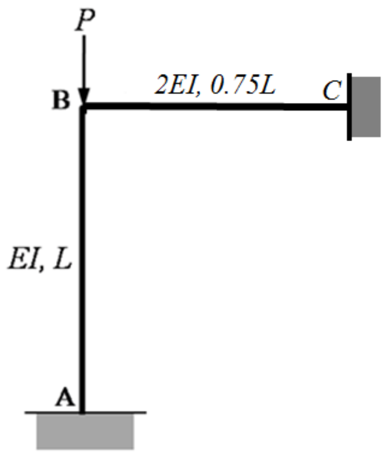


Figure 7

(b) Sketch the corresponding buckling mode for the braced frame shown in **Figure 8**. Subsequently, explain why critical load P_{cr} for columns in the braced frame lies within the following range :

$$2P_e < P_{cr} < 4P_e$$

where P_e : $\pi^2 EI/L^2$.

[8 marks]

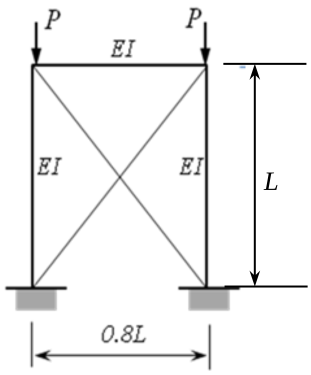


Figure 8

6. The conditions under which real member acts differ in many ways from the idealized conditions assumed in the analysis of the elastic buckling of a perfect member. Real member is not perfectly straight and their loads are applied eccentrically, while accidental transverse load may act. Explain complete with sketches, the real behaviour of the compression member in the design of steel structures.

[25 marks]

APPENDIX 1

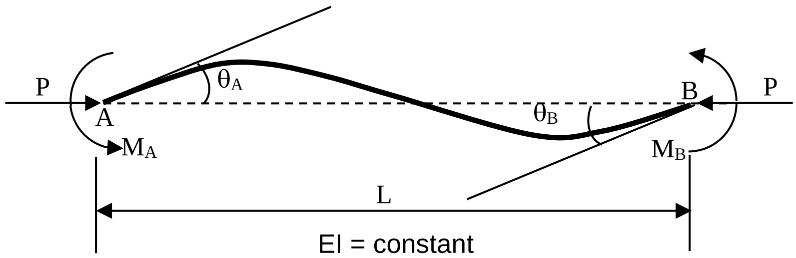
Slope deflection equations for a beam-column are given as follows :

$$M_A = \frac{EI}{L}\left(\alpha_n\theta_A + \alpha_f\theta_B\right)$$
$$M_B = \frac{EI}{L}\left(\alpha_f\theta_A + \alpha_n\theta_B\right)$$

where:

$$\alpha_n = \frac{\phi_n}{\phi_n^2 - \phi_f^2} \quad ;\alpha_f = \frac{\phi_f}{\phi_n^2 - \phi_f^2}$$
$$\phi_n = \frac{1}{(kL)^2}\left(1 - kL\cot kL\right) \qquad \phi_f = \frac{1}{(kL)^2}\left(kL\csc kL - 1\right)$$
$$k = \sqrt{\frac{P}{EI}}$$

and M_A, M_B, θ_A and θ_B are as shown in Fig.A1.



APPENDIX 2

General Solution for Equations of Motion

A) Response under free vibration:

i) Undamped SDOF system

$$u(t) = A\cos\omega_n t + B\sin\omega_n t$$

ii) Damped SDOF system

$$u(t) = e^{-\xi\omega_n t}\left[A\cos\omega_D t + B\sin\omega_D t\right]$$

B) Response to harmonic vibration, $F(t) = F_0 \sin\omega t$

i) Undamped SDOF system

$$u(t) = A\cos\omega_n t + B\sin\omega_n t + \frac{F_0}{k}\frac{1}{1-(\omega/\omega_n)^2}\sin\omega t$$

ii) Damped SDOF system

$$u(t) = e^{-\xi\omega_n t}(A\cos\omega_D t + B\sin\omega_D t) + C\sin\omega t + D\cos\omega t$$

$$C = \frac{F_0}{k}\frac{1-(\omega/\omega_n)^2}{\left[1-(\omega/\omega_n)^2\right]^2 + \left[2\xi(\omega/\omega_n)\right]^2}$$

$$D = \frac{F_0}{k}\frac{-2\xi(\omega/\omega_n)}{\left[1-(\omega/\omega_n)^2\right]^2 + \left[2\xi(\omega/\omega_n)\right]^2}$$

(C) Response to a block pulse

i) Undamped SDOF system

$$u(t) = A\cos\omega_n t + B\sin\omega_n t + \frac{F_0}{k}$$

ii) Damped SDOF system

$$u(t) = e^{-\xi\omega_n t}(A\cos\omega_D t + B\sin\omega_D t) + F_0/k$$

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